

# Fluid statics

Fluid statics is concerned with the balance of forces which stabilise fluids at rest. In the case of a liquid, as the pressure largely changes according to its height, it is necessary to take its depth into account. Furthermore, even in the case of relative rest (e.g. the case where the fluid is stable relative to its vessel even when the vessel is rotating at high speed), the fluid can be regarded as being at rest if the fluid movement is observed in terms of coordinates fixed upon the vessel.

## 3.1 Pressure

When a uniform pressure acts on a flat plate of area  $A$  and a force  $P$  pushes the plate, then

$$p = P/A \quad (3.1)$$

In this case,  $p$  is the pressure and  $P$  is the pressure force. When the pressure is not uniform, the pressure acting on the minute area  $\Delta A$  is expressed by the following equation:

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA} \quad (3.2)$$

### 3.1.1 Units of pressure

The unit of pressure is the pascal (Pa), but it is also expressed in bars or metres of water column (mH<sub>2</sub>O).<sup>1</sup> The conversion table of pressure units is given in Table 3.1. In addition, in some cases atmospheric pressure is used:

$$1 \text{ atm} = 760 \text{ mmHg (at } 273.15 \text{ K, } g = 9.806 \text{ 65 m/s}^2) = 101 \text{ 325 Pa} \quad (3.3)$$

1 atm is standard 1 atmospheric pressure in meteorology and is called the standard atmospheric pressure.

<sup>1</sup> Refer to the spread of 'aqua' at the end of this chapter (p. 37).

**Table 3.1** Conversion of pressure units

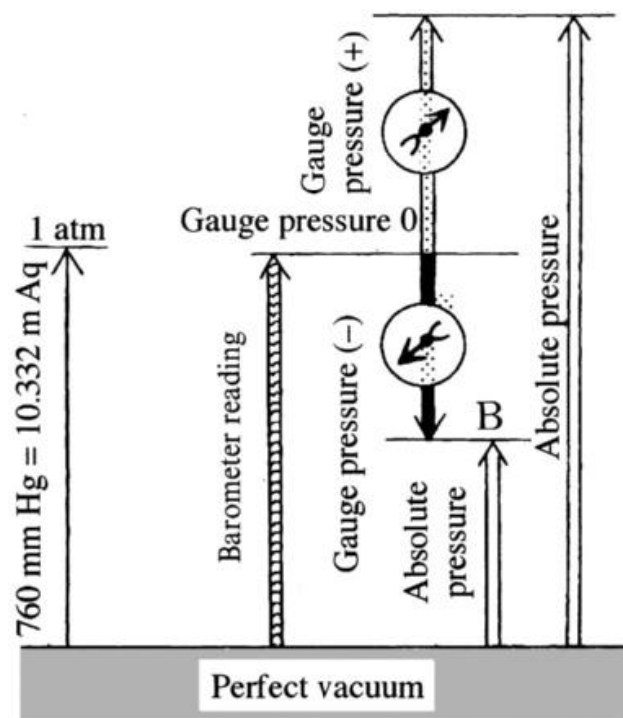
Name of unit	Unit	Conversion
Pascal	Pa	1 Pa = 1 N/m <sup>2</sup>
Bar	bar	1 bar = 0.1 MPa
Water column metre	mH <sub>2</sub> O	1 mH <sub>2</sub> O = 9 806.65 Pa
Atmospheric pressure	atm	1 atm = 101 325 Pa
Mercury column metre	mHg	1 mHg = 1/0.76 atm
Torr	torr	1 torr = 1 mm Hg

### 3.1.2 Absolute pressure and gauge pressure

There are two methods used to express the pressure: one is based on the perfect vacuum and the other on the atmospheric pressure. The former is called the absolute pressure and the latter is called the gauge pressure. Then,

$$\text{gauge pressure} = \text{absolute pressure} - \text{atmospheric pressure}$$

In gauge pressure, a pressure under 1 atmospheric pressure is expressed as a negative pressure. This relation is shown in Fig. 3.1. Most gauges are constructed to indicate the gauge pressure.

**Fig. 3.1** Absolute pressure and gauge pressure

### 3.1.3 Characteristics of pressure

The pressure has the following three characteristics.

1. The pressure of a fluid always acts perpendicular to the wall in contact with the fluid.
2. The values of the pressure acting at any point in a fluid at rest are equal regardless of its direction. Imagine a minute triangular prism of unit width in a fluid at rest as shown in Fig. 3.2. Let the pressure acting on the small surfaces  $dA_1$ ,  $dA_2$  and  $dA$  be  $p_1$ ,  $p_2$  and  $p$  respectively. The following equations are obtained from the balance of forces in the horizontal and vertical directions:

$$p_1 dA_1 = p dA \sin \theta$$

$$p_2 dA_2 = p dA \cos \theta + \frac{1}{2} dA_1 dA_2 \rho g$$

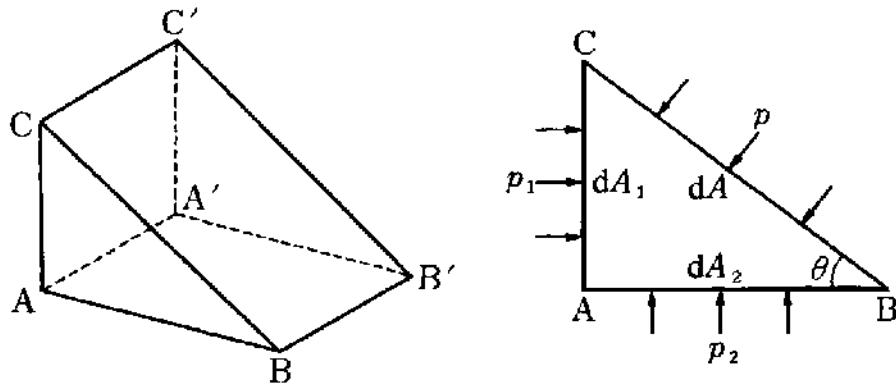


Fig. 3.2 Pressure acting on a minute triangular prism

The weight of the triangle pillar is doubly infinitesimal, so it is omitted. From geometry, the following equations are obtained:

$$dA \sin \theta = dA_1$$

$$dA \cos \theta = dA_2$$

Therefore, the following relation is obtained:

$$p_1 = p_2 = p \tag{3.4}$$

Since angle  $\theta$  can be given any value, values of the pressure acting at one point in a fluid at rest are equal regardless of its direction.

3. The fluid pressure applied to a fluid in a closed vessel is transmitted to all parts at the same pressure value as that applied (Pascal's law).

In Fig. 3.3, when the small piston of area  $A_1$  is acted upon by the force  $F_1$ , the liquid pressure  $p = F_1/A_1$  is produced and the large piston is acted upon by the force  $F_2 = pA_2$ . Thus

$$F_2 = F_1 \frac{A_2}{A_1} \tag{3.5}$$



### Blaise Pascal (1623–62)

French mathematician, physicist and philosopher. He had the ability of a highly gifted scientist even in early life, invented an arithmetic computer at 19 years old and discovered the principle of fluid mechanics that carries his name. Many units had appeared as the units of pressure, but it was decided to use the pascal in SI units in memory of his achievements.

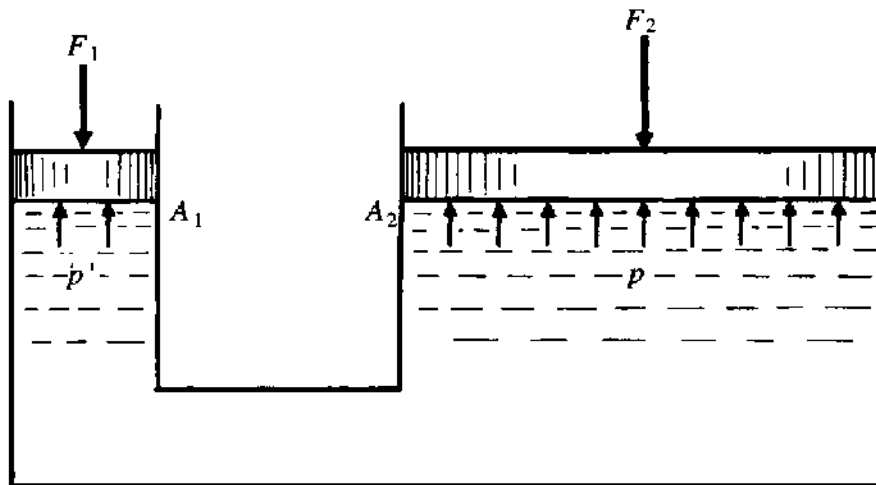


Fig. 3.3 Hydraulic press

So this device can create the large force  $F_2$  from the small force  $F_1$ . This is the principle of the hydraulic press.

### 3.1.4 Pressure of fluid at rest

In general, in a fluid at rest the pressure varies according to the depth. Consider a minute column in the fluid as shown in Fig. 3.4. Assume that the sectional area is  $dA$  and the pressure acting upward on the bottom surface is  $p$  and the pressure acting downward on the upper surface ( $dz$  above the bottom surface) is  $p + (dp/dz)dz$ . Then, from the balance of forces acting on the column, the following equation is obtained:

$$p dA - \left( p + \frac{dp}{dz} dz \right) dA - \rho g dA dz = 0$$

or

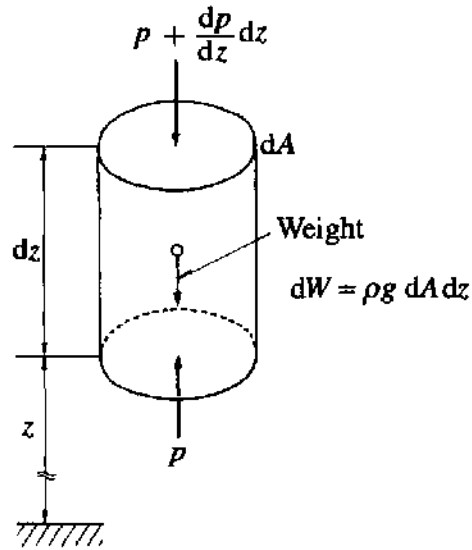


Fig. 3.4 Balance of vertical minute cylinder

$$\frac{dp}{dz} = -\rho g \tag{3.6}$$

Since  $\rho$  is constant for liquid, the following equation ensues:

$$p = -\rho g \int dz = -\rho g z + c \tag{3.7}$$

When the base point is set at  $z_0$  below the upper surface of liquid as shown in Fig. 3.5, and  $p_0$  is the pressure acting on that surface, then  $p = p_0$  when  $z = z_0$ , so

$$c = p_0 + \rho g z_0$$

Substituting this equation into eqn (3.7),

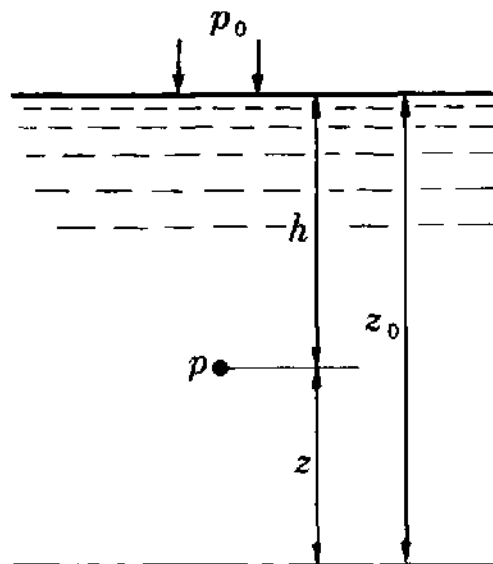


Fig. 3.5 Pressure in liquid

$$p = p_0 + (z_0 - z)\rho g = p_0 + \rho gh \quad (3.8)$$

Thus it is found that the pressure inside a liquid increases in proportion to the depth.

For the case of a gas, let us study the relation between the pressure and the height of the atmosphere surrounding the earth. In this case, since the density of gas changes with pressure, it is not possible to integrate simply as in the case of a liquid. As the altitude increases, the temperature decreases. Assuming this temperature change to be polytropic, then  $pv^n = \text{constant}$  is the defining relationship.

Putting the pressure and density at  $z = 0$  (sea level) as  $p_0$  and  $\rho_0$  respectively, then

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n} \quad (3.9)$$

Substituting  $\rho$  into eqn (3.6),

$$dz = -\frac{dp}{\rho g} = -\frac{1}{g} \frac{p_0^{1/n}}{\rho_0} p^{-1/n} dp = -\frac{1}{g} \frac{p_0}{\rho_0} \left(\frac{p_0}{p}\right)^{1/n} d\left(\frac{p}{p_0}\right) \quad (3.10)$$

Integrating this equation from  $z = 0$  (sea level),

$$z = \int_0^z dz = \frac{1}{g} \frac{n}{n-1} \frac{p_0}{\rho_0} \left[ 1 - \left(\frac{p}{p_0}\right)^{(n-1)/n} \right] \quad (3.11)$$

The relation between the height and the atmospheric pressure develops into the following equation by eqn (3.11):

$$\frac{p(z)}{p_0} = \left( 1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{n/(n-1)} \quad (3.12)$$

Also, the density is obtained as follows from eqs (3.9) and (3.12):

$$\frac{\rho(z)}{\rho_0} = \left( 1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{1/(n-1)} \quad (3.13)$$

When the absolute temperatures at sea level and at the point of height  $z$  are  $T_0$  and  $T$  respectively, the following equation is obtained from eqn (2.14):

$$\frac{p}{\rho T} = \frac{p_0}{\rho_0 T_0} = R \quad (3.14)$$

From eqs (3.12)–(3.14)

$$\frac{T(z)}{T_0} = 1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \quad (3.15)$$

From eqn (3.15)

$$\frac{dT}{dz} = -\frac{n-1}{n} \frac{\rho_0 g}{p_0} T_0 = -\frac{n-1}{n} \frac{g}{R} \quad (3.16)$$

In aeronautics, it has been agreed to make the combined values of  $p_0 = 101.325 \text{ kPa}$ ,  $T_0 = 288.15 \text{ K}$  and  $\rho_0 = 1.225 \text{ kg/m}^3$  the standard atmos-

pheric condition at sea level.<sup>2</sup> The temperature decreases by 0.65°C every 100 m of height in the troposphere up to approximately 1 km high, but is constant at -50.5°C from 1 km to 10 km high. For the troposphere, from the above values for  $p_0$ ,  $T_0$  and  $\rho_0$  in eqn (3.10),  $n = 1.235$  is obtained as the polytropic index.

### 3.1.5 Measurement of pressure

#### Manometer

A device which measures the fluid pressure by the height of a liquid column is called a manometer. For example, in the case of measuring the pressure of liquid flowing inside a pipe, the pressure  $p$  can be obtained by measuring the height of liquid  $H$  coming upwards into a manometer made to stand upright as shown in Fig. 3.6(a). When  $p_0$  is the atmospheric pressure and  $\rho$  is the density, the following equation is obtained:

$$p = p_0 + \rho gH \tag{3.17}$$

When the pressure  $p$  is large, this is inconvenient because  $H$  is too high. So a U-tube manometer, as shown in Fig. 3.6(b), containing a high-density liquid such as mercury is used. In this case, when the density is  $\rho'$ ,

$$p + \rho gH = p_0 + \rho' gH'$$

or

$$p = p_0 + \rho' gH' - \rho gH \tag{3.18}$$

In the case of measuring the air pressure,  $\rho' \gg \rho$ , so  $\rho gH$  in eqn (3.18) may be omitted. In the case of measuring the pressure difference between two pipes in both of which a liquid of density  $\rho$  flows, a differential manometer as

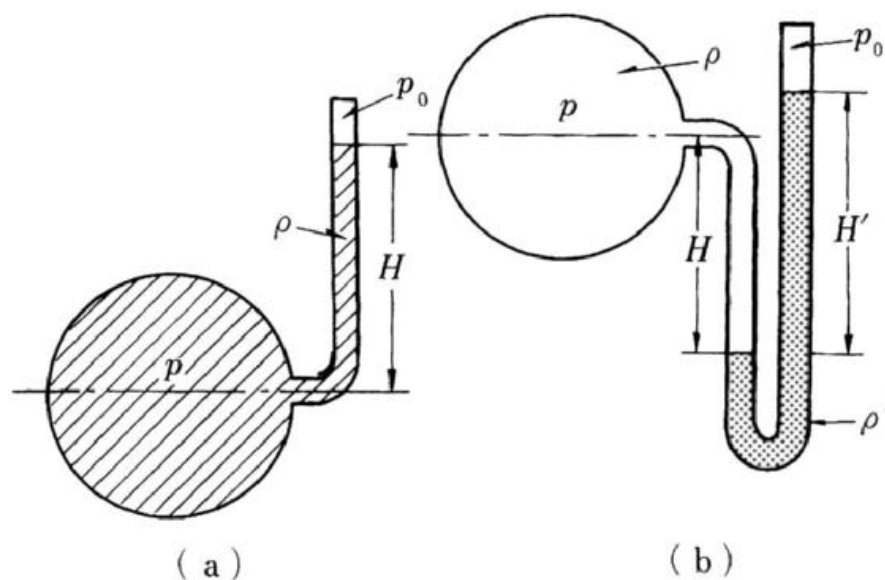


Fig. 3.6 Manometer

<sup>2</sup> ISO 2533-1975E.

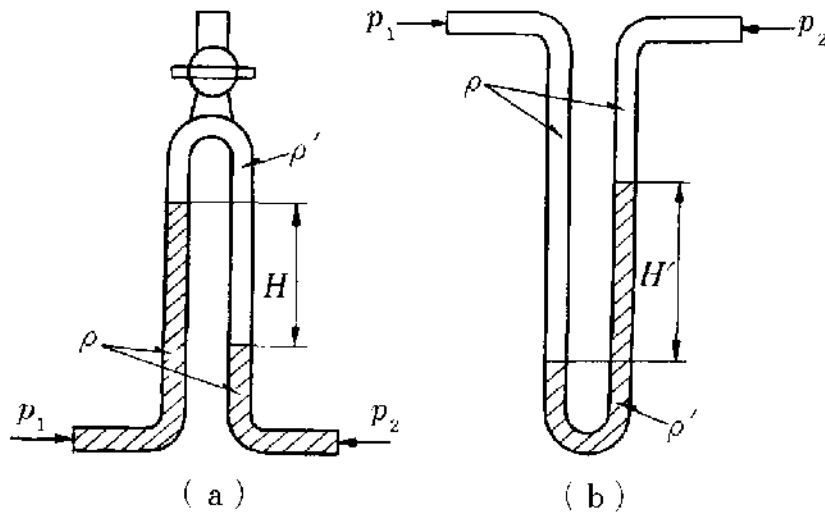


Fig. 3.7 Differential manometer (1)

shown in Fig. 3.7 is used. In the case of Fig. 3.7(a), where the differential pressure of the liquid is small, measurements are made by filling the upper section of the meter with a liquid whose density is less than that of the liquid to be measured, or with a gas. Thus

$$p_1 - p_2 = (\rho - \rho')gH \tag{3.19}$$

and in the case where  $\rho'$  is a gas,

$$\rho_1 - \rho_2 = \rho gH \tag{3.20}$$

Figure 3.7(b) shows the case when the differential pressure is large. This time, a liquid column of a larger density than the measuring fluid is used.

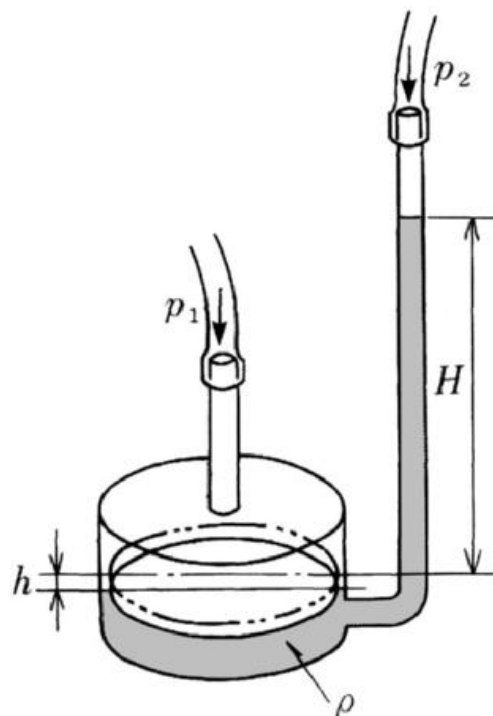


Fig. 3.8 Differential manometer (2)



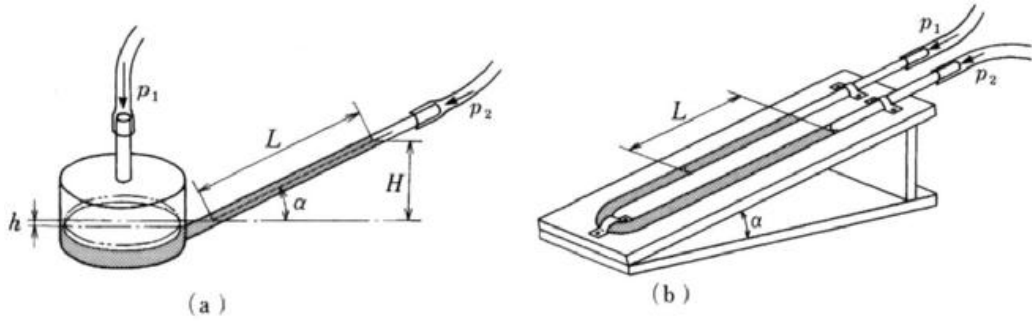


Fig. 3.9 Inclined manometer

Thus

$$p_1 - p_2 = (\rho' - \rho)gH' \tag{3.21}$$

and in the case where  $\rho$  is a gas,

$$p_1 - p_2 = \rho'gH' \tag{3.22}$$

A U-tube manometer as shown in Fig. 3.7 is inconvenient for measuring fluctuating pressure, because it is necessary to read both the right and left water levels simultaneously to measure the different pressure. For measuring the differential pressure, if the sectional area of one tube is made large enough, as shown in Fig. 3.8, the water column of height  $H$  could be measured by just reading the liquid surface level in the other tube because the surface fluctuation of liquid in the tank can be ignored.

To measure a minute pressure, a glass tube inclined at an appropriate angle as shown in Fig. 3.9 is used as an inclined manometer. When the angle of inclination is  $\alpha$  and the movement of the liquid surface level is  $L$ , the differential pressure  $H$  is as shown in the following equation:

$$H = L \sin \alpha \tag{3.23}$$

Accordingly, if  $\alpha$  is made smaller, the reading of the pressure is magnified. Besides this, Göttingen-type micromanometer, Chattock tilting micromanometer, etc., are used.

**Elastic-type pressure gauge**

An elastic-type pressure gauge is a type of pressure gauge which measures the pressure by balancing the pressure of the fluid with the force of deformation of an elastic solid. The Bourdon tube (invented by Eugene Bourdon, 1808–84) (Fig. 3.10), the diaphragm (Fig. 3.11), the bellows, etc., are widely employed for this type of pressure gauge.

Of these, the Bourdon tube pressure gauge (Bourdon gauge) of Fig. 3.10 is the most widely used in industry. A curved metallic tube of elliptical cross-section (Bourdon tube) is closed at one end which is free to move, but the other end is rigidly fixed to the frame. When the pressure enters from the fixed end, the cross-section tends to become circular so the free end moves outward. By amplifying this movement, the pressure values can be read. When the pressure becomes less than the atmospheric pressure (vacuum), the free end moves inward, so this gauge can be used as a vacuum gauge.

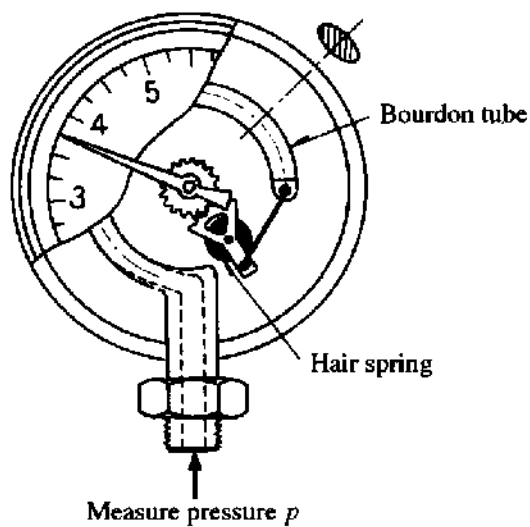


Fig. 3.10 Bourdon tube pressure gauge

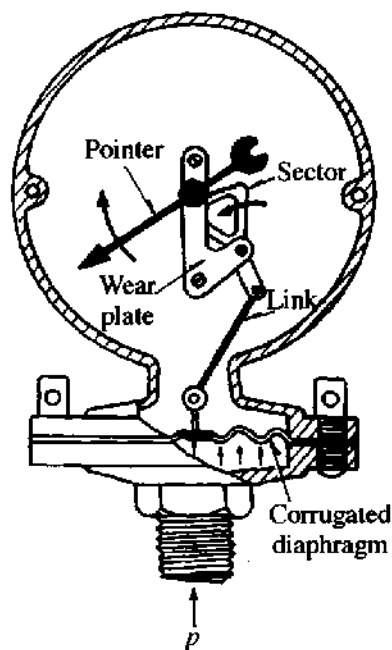


Fig. 3.11 Diaphragm pressure gauge

**Electric-type pressure gauge**

The pressure is converted to the force or displacement passing through the diaphragm, Bourdon tube bellows, etc., and is detected as a change in an electrical property using a wire strain gauge, a semiconductor strain gauge



	PGM-C	PE-J, P
	(a) Small size	(b) For indicating pressure of an engine
Pressure range (MPa)	0.2–1.0	5–20
Natural frequency (kHz)	25–40	25–40
Response frequency (kHz)	5–8	5–8

Fig. 3.12 Wire strain gauge type of pressure transducer

(applied piezoresistance effect), etc. These types of pressure gauge are useful for measuring fluctuating pressures. Two examples of pressure gauges utilising the wire strain gauge are shown in Fig. 3.12.

## 3.2 Forces acting on the vessel of liquid

How large is the force acting on the whole face of a solid wall subject to water pressure, such as the bank of a dam, the sluice gate of a dam or the wall of a water tank? How large must the torque be to open the sluice gate of a dam? What is the force required to tear open a cylindrical vessel subject to inside pressure? Here, we will study forces like these.

### 3.2.1 Water pressure acting on a bank or a sluice gate

How large is the total force due to the water pressure acting on a bank built at an angle  $\theta$  to the water surface (Fig. 3.13)? Here, disregarding the atmospheric pressure, the pressure acting on the surface is zero. The total pressure  $dP$  acting on a minute area  $dA$  is  $\rho gh dA = \rho g y \sin \theta dA$ . So, the total pressure  $P$  acting on the under water area of the bank wall  $A$  is:

$$P = \int_A dP = \rho g \sin \theta \int_A y dA$$

When the centroid<sup>3</sup> of  $A$  is  $G$ , its  $y$  coordinate is  $y_G$  and the depth to  $G$  is  $h_G$ ,  $\int_A y dA = y_G A$ . So the following equation is obtained:

$$P = \rho g \sin \theta y_G A = \rho g h_G A \quad (3.24)$$

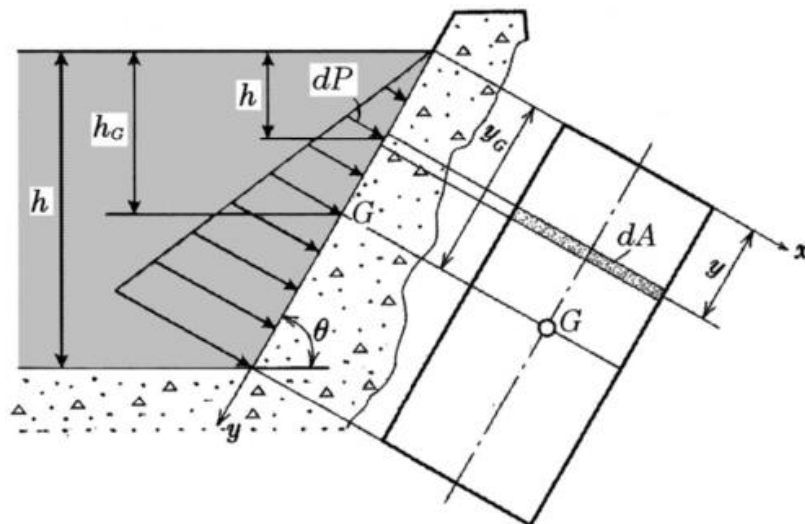
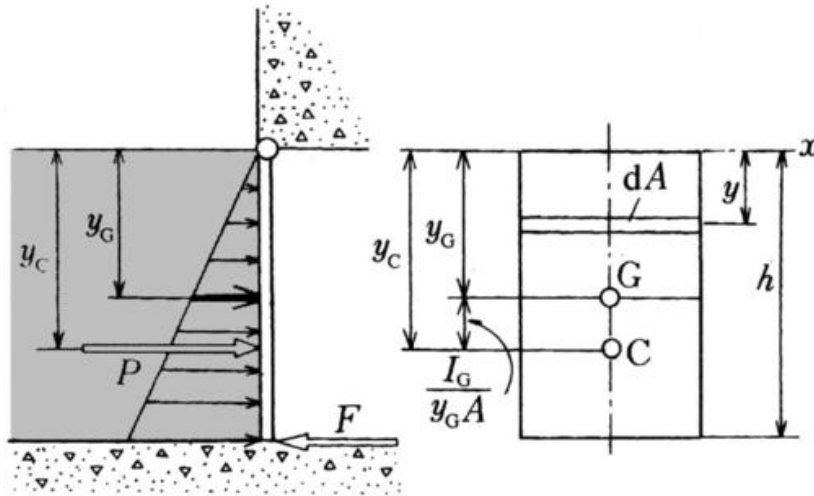


Fig. 3.13 Force acting on dam

<sup>3</sup> The centre of mass when the mass is distributed uniformly on the plane of some figure, namely the point applied to the centre of gravity, is called a centroid.



**Fig. 3.14** Revolving power acting on water gate (1) (case where revolving axis of water gate is just on the water level)

So the total force  $P$  equals the product of the pressure at the centroid  $G$  and the underwater area of the bank wall.

Next, let us study a rectangular sluice gate as shown in Fig. 3.14. How large is the torque acting on its turning axis (the  $x$  axis)? The force  $P$  acting on the whole plane of the gate is  $\rho g y_G A$  by eqn (3.24). The force acting on a minute area  $dA$  (a horizontal strip of the gate face) is  $\rho g y dA$ , the moment of this force around the  $x$  axis is  $\rho g y dA \times y$  and the total moment on the gate is  $\int \rho g y^2 dA = \rho g \int y^2 dA$ .  $\int y^2 dA$  is called the geometrical moment of inertia  $I_x$  for the  $x$  axis.

Now let us locate the action point of  $P$  (i.e. the centre of pressure  $C$ ) at which a single force  $P$  produces a moment equal to the total sum of the moments around the turning axis ( $x$  axis) of the sluice gate produced by the total water pressure acting on all points of the gate. When the location of  $C$  is  $y_C$ ,

$$P y_C = \rho g I_x \quad (3.25)$$

Now, when  $I_G$  is the geometrical moment of inertia of area for the axis which is parallel to the  $x$  axis and passes through the centroid  $G$ , the following relation exists:<sup>4</sup>

$$I_x = I_G + A y_G^2 \quad (3.26)$$

Values of  $I_G$  for a rectangular plate and for a circular plate are shown in Fig. 3.15.

Substitute eqn (3.26) into (3.25) to calculate  $y_C$

$$y_C = y_G + \frac{I_G}{A y_G} = y_G + \frac{h^2}{12 y_G} \quad (3.27)$$

<sup>4</sup> *Parallel axis theorem:* The moment of inertia with respect to any axis equals the sum of the moment of inertia with respect to the axis parallel to this axis which passes through the centroid and the product of the sectional area and the square of the distance to the centroid from the former axis.

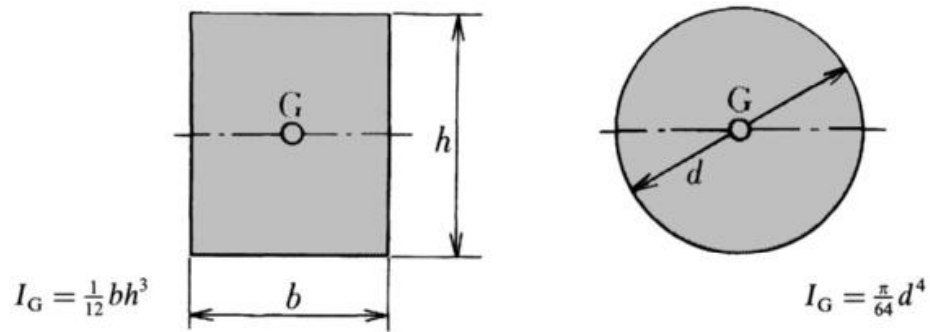


Fig. 3.15 Geometrical moment of inertia for axis passing centroid G

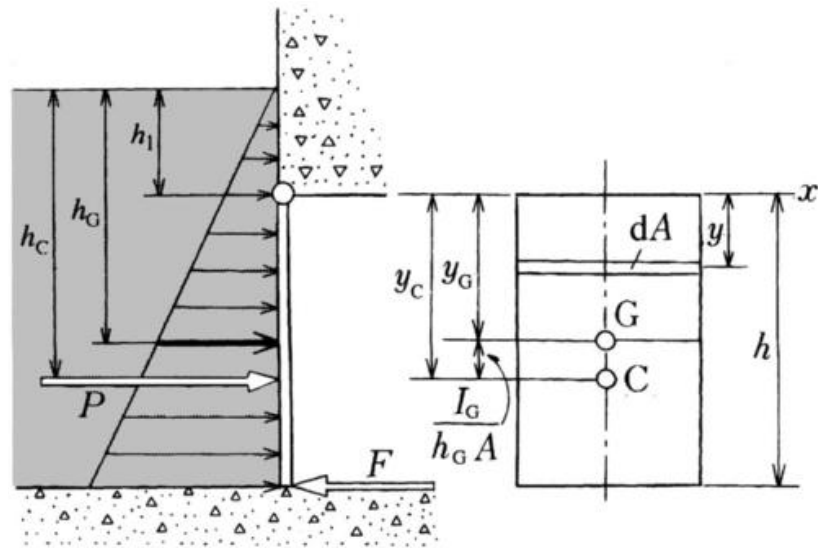


Fig. 3.16 Rotational force acting on water gate (2) (case where water gate is under water)

From eqn (3.27), it is clear that the action point C of the total pressure  $P$  is located deeper than the centroid  $G$  by  $h^2/12y_G$ .

The position of  $y_C$  in such a case where the sluice gate is located under the water surface as shown in Fig. 3.16 is given by eqn (3.28) where  $h_G$  is substituted for  $y_G$  in the second term on the right of eqn (3.27):

$$y_C = y_G + \frac{h^2}{12h_G} \tag{3.28}$$

### 3.2.2 Force to tear a cylinder

In the case of a thin cylinder where the inside pressure is acting outward, as shown in Fig. 3.17(a), what kind of force is required to tear this cylinder in the longitudinal direction? Now, consider the cylinder longitudinally half sectioned as shown in Fig. 3.17(b), with diameter  $d$ , length  $l$  and inside pressure  $p$ . The force acting on the assumed vertical centre wall ABCD is  $pdl$  which balances the force in the  $x$  direction acting outward on the cylinder wall. In other words, the force generated by the pressure in the  $x$  direction on a curved surface equals the pressure  $pdl$ , since the same pressure acts on the projected area of the curved surface. Furthermore, this force is the force  $2Tl$

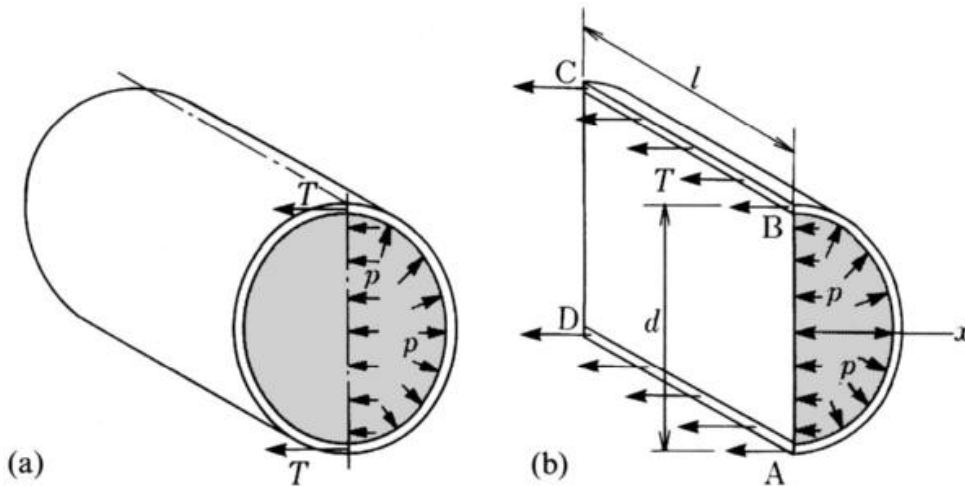


Fig. 3.17 Cylinder acted on by inertial pressure

( $T$  is the force acting per unit length of wall which tears this cylinder longitudinally in halves along the lines  $BC$  and  $AD$ ):

$$2Tl = pdl$$

or

$$T = pd/2 \quad (3.29)$$

If the tensile stress due to  $T$  is lower than the allowable stress, safety is assured. By utilising this principle, a thin-walled pressure tank can be designed.

### 3.3 Why does a ship float?

Fluid pressure acts all over the wetted surface of a body floating in a fluid, and the resultant pressure acts in a vertical upward direction. This force is called buoyancy. The buoyancy of air is small compared with the gravitational force of the immersed body, so it is normally ignored.

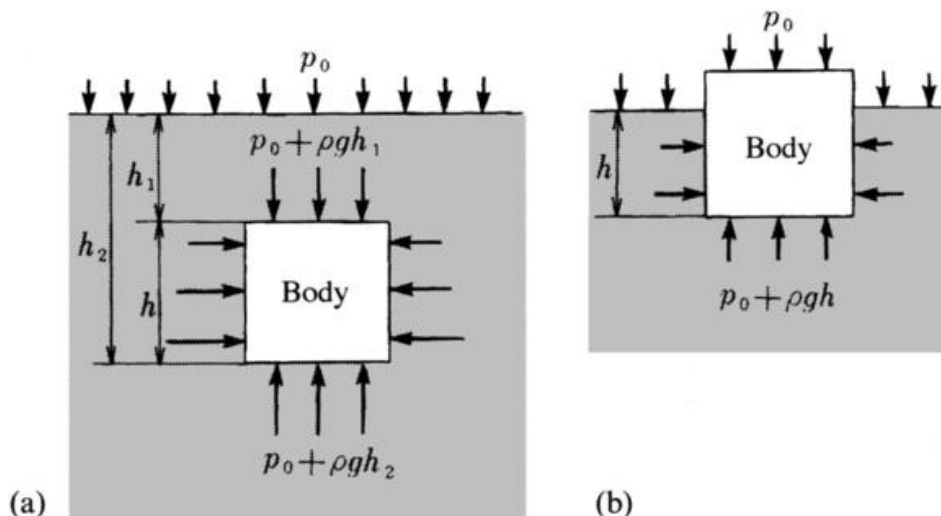


Fig. 3.18 Cube in liquid

**Archimedes (287–212bc)**

The greatest mathematician, physicist and engineer in ancient Greece, and the discoverer of the famous 'Principle of Archimedes'. Archimedes received guidance in astronomy from his father, an astronomer, and made astronomical observations since his early days. He invented a planetarium turned by hydropower and a screw pump. He carried out research in solid and fluid dynamics as well as on the lever, the centre of gravity and buoyancy. Archimedes was one of those scientists who are talented in both theory and practice.



Suppose that a cube is located in a liquid of density  $\rho$  as shown in Fig. 3.18. The pressure acting on the cube due to the liquid in the horizontal direction is balanced right and left. For the vertical direction, where the atmospheric pressure is  $p_0$ , the force  $F_1$  acting on the upper surface  $A$  is expressed by the following equation:

$$F_1 = (p_0 + \rho gh_1)A \quad (3.30)$$

The force  $F_2$  acting on the lower surface is

$$F_2 = (p_0 + \rho gh_2)A \quad (3.31)$$

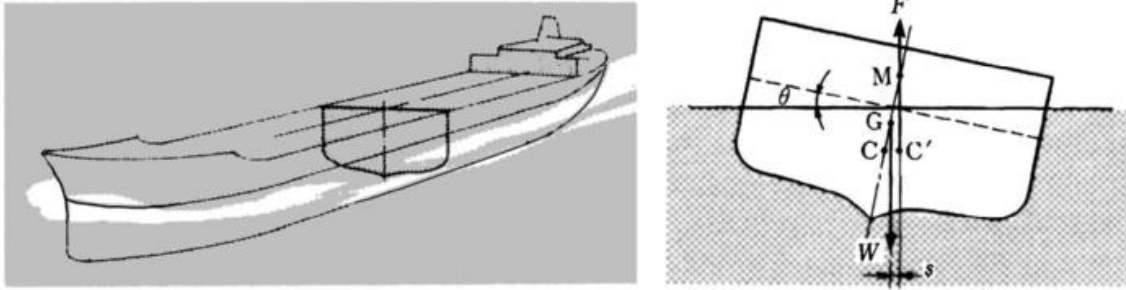
So, when the volume of the body in the liquid is  $V$ , the resultant force  $F$  from the pressure acting on the whole surface of the body is

$$F = F_2 - F_1 = \rho g(h_2 - h_1)A = \rho ghA = \rho gV \quad (3.32)$$

The same applies to the case where a cube is floating as shown in Fig. 3.18(b). From this equation, the body in the liquid experiences a buoyancy equal to the weight of the liquid displaced by the body. This result is known as Archimedes' principle. The centre of gravity of the displaced liquid is called 'centre of buoyancy' and is the point of action of the buoyancy force.

Next, let us study the stability of a ship. Figure 3.19 shows a ship of weight  $W$  floating in the water with an inclination of small angle  $\theta$ . The location of the centroid  $G$  does not change with the inclination of the ship. But since the centre of buoyancy  $C$  moves to the new point  $C'$ , a couple of forces  $Ws = Fs$  is produced and this couple restores the ship's position to stability.

The forces of the couple  $Ws$  are called restoring forces. The intersecting point  $M$  on the vertical line passing through the centre of buoyancy  $C'$  (action line of the buoyancy  $F$ ) and the centre line of the ship is called the



**Fig. 3.19** Stability of a ship

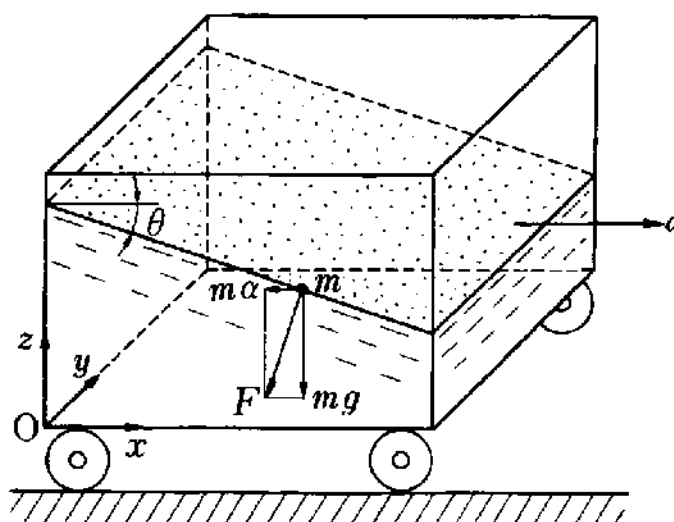
metacentre, and  $GM$  is called the metacentric height.<sup>5</sup> As shown in the figure, if  $M$  is located higher than  $G$ , the restoring force acts to stabilise the ship, but if  $M$  is located lower than  $G$ , the couple of forces acts to increase the roll of the ship and so make the ship unstable.

### 3.4 Relatively stationary state

When a vessel containing a liquid moves in a straight line or rotates, if there is no relative flow of the liquid while the vessel and liquid move as a body, it is possible to treat this as the mechanics of a stationary state. This state is called a relatively stationary state.

#### 3.4.1 Equiaccelerated straight-line motion

Suppose that a vessel filled with liquid is moving in a straight line at constant acceleration on the horizontal level as shown in Fig. 3.20. Further consider a minute element of mass  $m$  on the liquid surface, where its acceleration is  $\alpha$ , the



**Fig. 3.20** Uniform accelerating straight-line motion

<sup>5</sup> How high is the metacentre of a real ship? It is said that the height of metacentre of a warship is about 0.8–1.2 m, a sailing ship 1.0–1.4 m and a large passenger ship 0.3–0.7 m. When these ships go out to sea the wave cycle is 12–13 seconds.



forces acting on  $m$  are gravity in a vertical downward direction  $-mg$ , and the inertial force in the reverse direction to the direction of acceleration  $-m\alpha$ .

There can be no force component normal to the direction of  $F$ , the resultant force of gravity and the inertial forces. Therefore, the pressure must be constant on the plane normal to the direction of  $F$ . In other words, this plane identifies the equipressure free surface.

When  $\theta$  is the angle formed by the free surface and the  $x$  direction, the following relation is easily obtained:

$$\tan \theta = \alpha/g \quad (3.33)$$

If  $h$  is the depth measured in the vertical direction to the free surface, the acceleration in this direction is  $\beta = F/m$ . Therefore,

$$p = \rho\beta h \quad (3.34)$$

This is the same relation as the stationary state.

### 3.4.2 Rotational motion

Let us study the height of the water surface in the case where a cylindrical vessel filled with liquid is rotating at constant angular velocity  $\omega$ . The movement at constant angular velocity like this is sometimes called gyrostatics, where the liquid surface poses a concave free surface. Then let us take cylindrical coordinates  $(r, \theta, z)$  as shown in Fig. 3.21. Consider a minute element of mass  $m$  on the equipressure plane. The forces acting on it are  $-mg$  due to the gravitational acceleration  $g$  in the vertical direction and  $-mr\omega^2$  due to the centripetal acceleration  $r\omega^2$  in the horizontal direction.

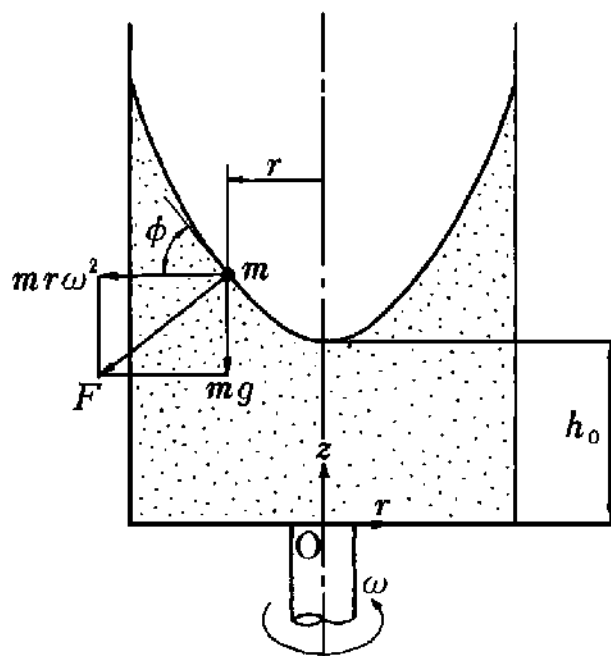


Fig. 3.21 Rotational motion around vertical axis

Since the vessel and liquid move in a body and the liquid stays in a relatively stationary state, the resultant force  $F$  is vertical to the free surface as in the previous case. If  $\phi$  is the angle formed by the free surface and the horizontal direction,

$$\tan \phi = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g} \quad (3.35)$$

but also

$$\tan \phi = \frac{dz}{dr}$$

Therefore,

$$\frac{dz}{dr} = \frac{r\omega^2}{g}$$

Putting  $c$  as a constant of integration,

$$z = \frac{\omega}{2g} r^2 + c \quad (3.36)$$

If  $z = h_0$  at  $r = 0$ ,  $c = h_0$  and the following equation is obtained from eqn (3.36):

$$z - h_0 = \frac{\omega^2 r^2}{2g} \quad (3.37)$$

The free surface is now a rotating parabolic surface.

### ***Spread of aqua***

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Aqua means water in Sanskrit, especially water offered to Buddha. The Italian word for water is *aqua*, and the Spanish is *agua*, both of which have the same etymological origin. Also, 'aqualung' is diving gear meaning water lung.

The Japanese word *aka* appeared in Japanese classics in the tenth, eleventh and thirteenth centuries. Furthermore, *aka* also means bilge water.

It is very interesting to know that Sanskrit *aqua* spread from India to Europe along the Silk Road and to Japan via China.

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### 3.5 Problems

1. What is the water pressure on the sea bottom at a depth of 6500 m? The specific gravity of sea water is assumed to be 1.03.
2. Obtain the pressure  $p$  at point A in Figs 3.22(a), (b) and (c).

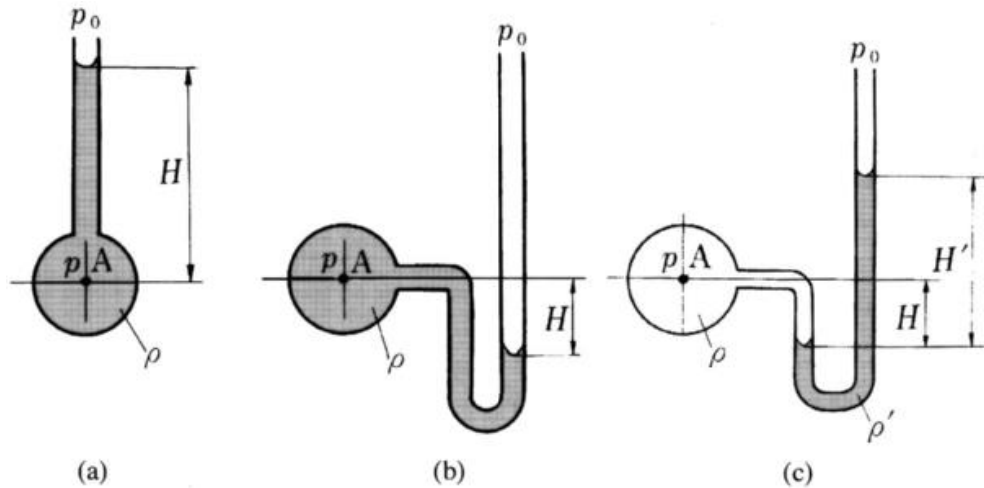


Fig. 3.22

3. Obtain the pressure difference  $p_1 - p_2$  in Figs 3.23(a) and (b).

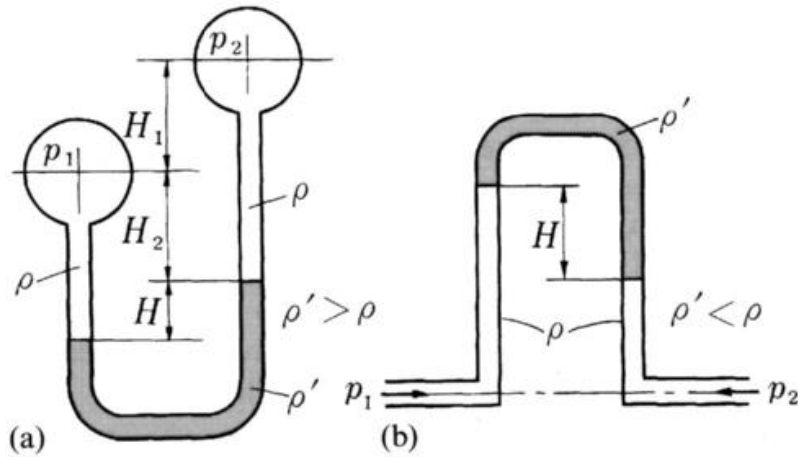


Fig. 3.23

4. On the inclined manometer in Fig. 3.24, whenever  $h$  changes by 1 mm, how high (in mm) is  $H$ ? (Sectional area  $A = 100a$  and  $\alpha = 30^\circ$ .)

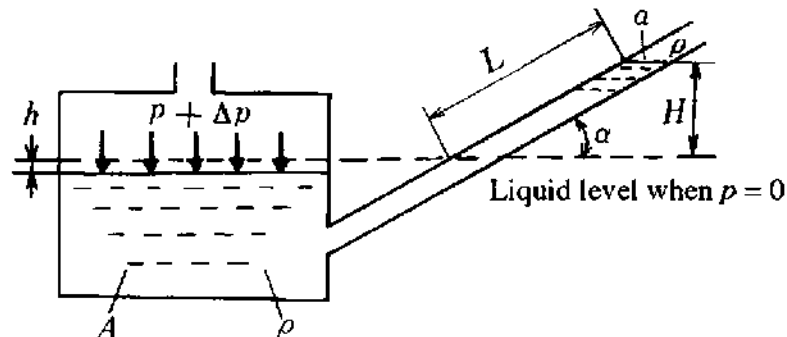


Fig. 3.24

5. In the case shown in Fig. 3.25, an oblong board 3 m high and 5 m wide is placed vertically in water in such a manner that its upper face is 5 m deep. Obtain the force acting on this board and the location of the centre of pressure.
6. What are the respective forces  $F$  acting on the lower stays of the water gates in Figs 3.14 and 3.16, provided that the height  $b$  of the water gate is 3 m, the width is 1 m, and  $h_1$  in Fig. 3.16 is 2 m?
7. A water gate 2 m high and 1 m wide is shown in Fig. 3.26. What is the force acting on the lower stay?
8. What is the force acting on a unit width of the dam wall shown in Fig. 3.27, if the water is 15 m deep and the wall is inclined at  $60^\circ$ ? Furthermore, how far along the wall from the water surface is the action point of the force?

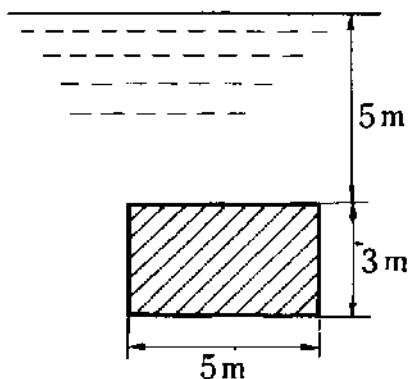


Fig. 3.25

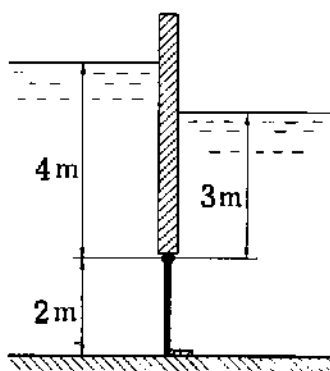


Fig. 3.26

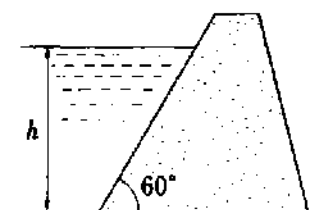


Fig. 3.27

9. As shown in Fig. 3.28, a circular water gate, diameter 2 m, is supported by a horizontal shaft. What is the moment around the shaft to keep the water gate closed?
10. A circular segment water gate, 5 m long, is set as shown in Fig. 3.29. Water is stored up to the upper face of the water gate. Obtain the magnitudes of the horizontal and vertical components of force and also the magnitude and the direction of the resultant force acting on this water gate.

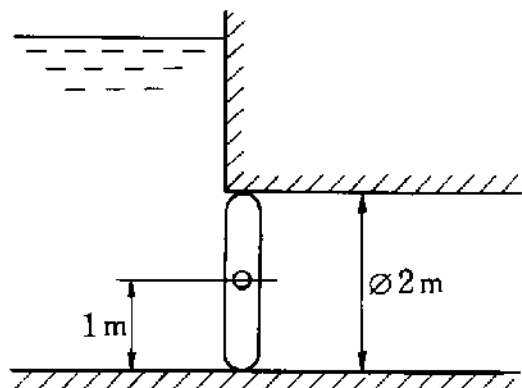


Fig. 3.28

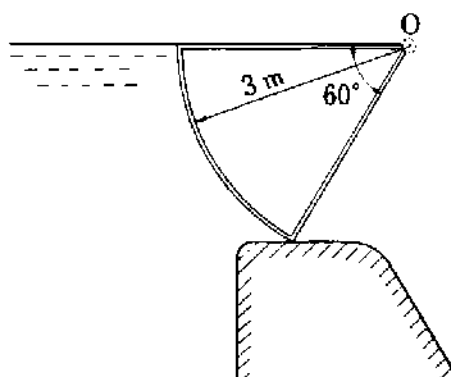


Fig. 3.29

11. An iceberg of specific gravity 0.92 is floating on the sea with a specific gravity of 1.025. If the volume of the iceberg above the water level is  $100 \text{ m}^3$ , what is the total volume of the iceberg?
12. As shown in Fig. 3.30 a body of specific gravity 0.8 is floating on the water. Obtain the height of its metacentre and the period of vibration whenever its side A is perturbed.<sup>6</sup> The effect of the additional mass of water can be omitted.

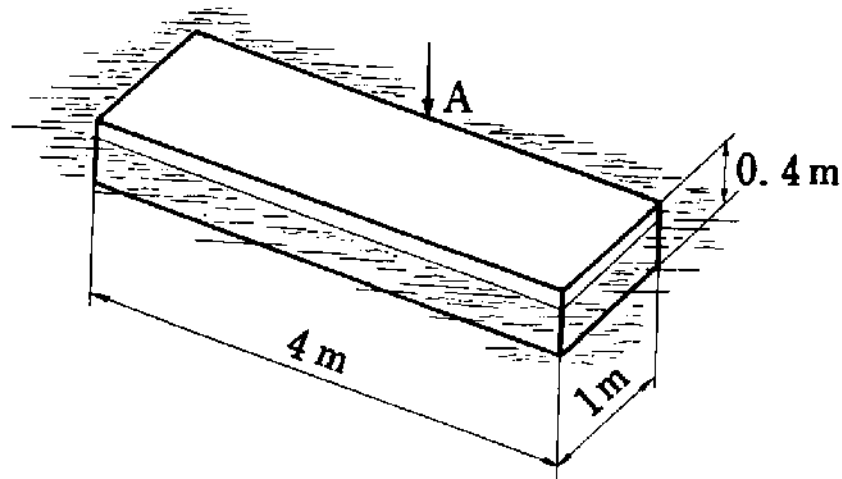


Fig. 3.30

13. A cylindrical vessel of radius  $r_0$  filled with water to height  $h$  is rotated around the central axis, and the difference in height of water level is  $h'$ . What is the rotational angular velocity? Furthermore, assuming  $r_0 = 10 \text{ cm}$  and  $h = 18 \text{ cm}$ , obtain  $\omega$  when  $h' = 10 \text{ cm}$  and also the number of revolutions per minute  $n$  when the cylinder bottom begins to appear.

<sup>6</sup> If  $h$  is the height of the metacentre,  $V$  the displacement volume,  $I$  the sectional secondary moment around the centre line of the water plane, and  $e$  the height from the centre of buoyancy to the centre of gravity, then

$$h = (I/V) - e$$

If  $J$  is the moment of inertia around the longitudinal axis passing through the centre of gravity,  $\theta$  the inclination angle and  $m$  the mass, the movement equation for crosswise vibration is (whenever  $\theta$  is small), then

$$J \frac{d^2\theta}{dt^2} = -mgh\theta$$

If  $T$  is the period,  $k = \sqrt{J/m}$  the turning radius around the centre of gravity, then

$$T = 2\pi \frac{k}{\sqrt{gh}}$$