Cracow University of Technology Institute of Thermal and Process Engineering **Division of Fluid Mechanics**	Name and surname:			Number of classes: **16**
Subject of classes: Identification of the fluid flow rate				Accademic year:
Date of execution:	Date of evaluation:	Mark:	Remarks:	

1. Measurements of air flow rate using Venturi tube

Volume flow rate can be calculated using following formula:

$$Q_z = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon_1 \cdot \frac{\pi}{4} \cdot d^2 \cdot \sqrt{\frac{2 \cdot \Delta p_z}{\rho}} \tag{1}$$

where:

β= d/D – Venturi tube modulus,

p_1 – total static pressure of flow in front of Venturi tube; $p_1 = p_b + p_{w1}$,

p_b – barometric pressure [Pa],

ρ - density of air – dependent on thermodynamic conditions of flow $\rho = \rho(p_1, T)$.

$$\varepsilon_1 = 1 - \left(0,41 + 0,35 \cdot \beta^4\right)\frac{\Delta p_z}{1,4 p_1} \tag{2}$$

Air density in flow conditions can be assign by application of the ideal gas equation of state – the Clapeyron equation:

$$\rho = \frac{p_1}{RT} \tag{3}$$

where:

R= 287 [J/kgK]- individual gas constant for air,

T – absolute temperature of air [K].

Flow correction factor C = C(β, Re) can be assigned using Stolz formula:

$$\begin{aligned} C = {} & 0,5959 + 0,0312 \cdot \beta^{2,1} - 0,184 \cdot \beta^8 + 0,0029 \cdot \beta^{2,5}\left(\frac{10^6}{\mathrm{Re}}\right)^{0,75} + \\ & + 0,09 \cdot \frac{25,4mm}{D} \cdot \beta^4 \cdot \left(1-\beta^4\right)^{-1} - 0,0337 \cdot \frac{25,4mm}{D} \cdot \beta^3 \end{aligned} \tag{4}$$

Due to the fact that Reynolds number is unknown, coefficient C must be assigned using iteration method. In first iteration in this method value of C is calculated for some arbitrary chosen value of Reynolds number Re = 50 000. For self-check – approximated value C in laboratory measurement condition is equal to C = 0,628. Reynolds number value in further iterations shall be assign using well known formula:

$$\mathrm{Re} = \frac{4 Q_z \rho}{\pi D \eta} \tag{5}$$

Dynamic viscosity of air η is a function of temperature and can be assign using Sutherland's viscosity law for the ideal gas:

$$\eta = \eta_0 \frac{273,15 + C_s}{T + C_s}\left(\frac{T}{273,15}\right)^{\frac{3}{2}} \tag{6}$$

where:

η_0 – dynamic viscosity in temperature equal to 273 K (for air $\eta_0 = 17,08 \cdot 10^{-6}$ [Pa·s])

C_s – Sutherland constant (for air $C_s = 112$)

Table 1. Measurement results obtained using Venturi tube

p_b = …………………….. [Pa]				d = 35·mm		D = 46·mm		β = …………….		
No.	Fan frequency	p_{w1} [mmH$_2$O]	Δp_z [mbar]	t [°C]	ρ [kg/m^3]	η [N·s/m^2]	ε_1	Re	C	Q_z [m^3/s]
1	…….Hz									
2	…….Hz									
3	…….Hz									
4	…….Hz									
5	…….Hz									

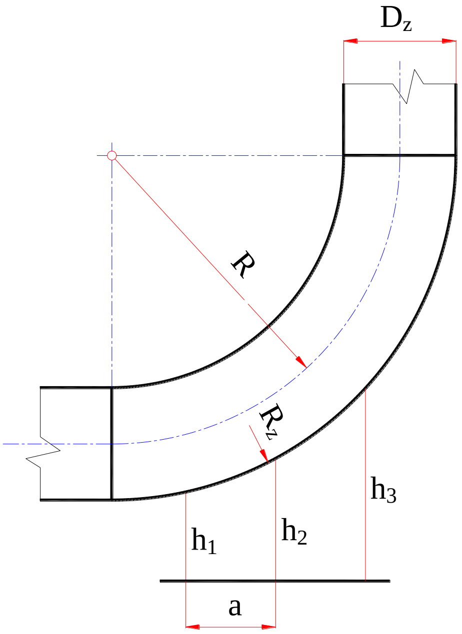
2. Measurements of air flow rate using elbow flowmeter (centrifugal-head flowmeter)

Shift a [mm]	Height			b = h$_2$ – h$_1$ [mm]	c = h$_3$ – h$_1$ [mm]	Measured radius R [mm]	Real radius R [mm]
	h$_1$ [mm]	h$_2$[mm]	h$_3$[mm]				
20							93,5±3

The elbow bending radius R can be assign using equation (7):

$$R = \frac{\sqrt{\left(4\cdot a^2 + c^2\right)\left(a^2 + b^2\right)\left(a^2 + c^2 - 2\cdot c\cdot b + b^2\right)}}{2\cdot a\cdot\left(c - 2\cdot b\right)} - \frac{D_z}{2} \quad (7)$$

where: D_z = 50 mm – outer pipe diameter .



Volumetric flow rate of air inside the pipeline can be assigned using formula (8):

$$Q_e = KA\sqrt{\frac{R}{D}\frac{\Delta p_e}{\rho}} \quad (8)$$

where:

$$K = 1 - \frac{6,5}{\sqrt{\mathrm{Re}}} \text{ - flow correction factor} \quad (9)$$

$$A = \frac{\pi D^2}{4} \quad (10)$$

$$\mathrm{Re} = \frac{4 Q_e \rho}{\pi D \eta} \quad (11)$$

In first iteration value of K is calculated for some arbitrary chosen value of Reynolds number, e.g. Re = 50 000. Then, Reynolds number value in further iterations shall be assign using formula (11).

Table 2. Measurement results obtained using elbow flowmeter

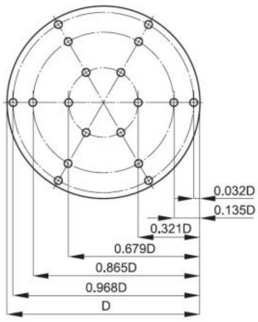
No.	Fan frequency	Δp_e [mbar]	Δp_e [Pa]	Re	K	Q_e [m^3/s]
1	………Hz					
2	………Hz					
3	……..…Hz					
4	……..…Hz					
5	……..…Hz					

3. Measurements of air flow rate using impact tube (Prandtl tube)

Table 3. Local flow velocity measure using Prandtl impact tube

No.	Fan frequency	v [m/s]			v_{av} [m/s]
		0,032 D =…….. [mm]	0,135 D =…….. [mm]	0,321 D =……...[mm]	
1	……… Hz				
2	……… Hz				
3	……… Hz				
4	……… Hz				
5	……… Hz				

Average velocity of air for each measurment series are calculated using eq. (12):



$$v_{av} = \frac{1}{3}\sum_{i=1}^{3} v_i \qquad (12)$$

Value of flow rate Q_p and Reynolds number Re are described by following formulas:

$$Q_P = v_{av}\,\frac{\pi D^2}{4} \qquad (13)$$

$$\mathrm{Re} = \frac{D\,\rho v_{av}}{\eta} \qquad (14)$$

Table 4. Measurement results obtained using Prandtl impact tube

No.	Fan frequency	v_{av} [m/s]	Re	Q_P [m^3/s]
1	………Hz			
2	…..…..Hz			
3	…..…..Hz			
4	…..…..Hz			
5	…..…..Hz			

4. Comparison of flow rate measurement results

Assuming, that volumetric flow rate measurement performed by Venturi tube is a reference result, assign appropriate relative deviation for elbow flowmeter and Prandtl tube using eq. (15) and (16):

$$\delta Q_k = \frac{Q_z - Q_e}{Q_z}\cdot 100\ [\%] \qquad (15)$$

$$\delta Q_p = \frac{Q_z - Q_p}{Q_z}\cdot 100\ [\%] \qquad (16)$$

Table 5. Comparison study results

No.	Fan frequency	δQ_e [%]	δQ_P [%]
1	………Hz		
2	………Hz		
3	…..…..Hz		
4	…..…..Hz		
5	…..…..Hz		